Brief information about the project

Name of the project	AP09058430 «Development of numerical methods for solving Navier-Stokes equations combining fictitious domains and conjugate equations» (0121PK00366)
Relevance	This project is dedicated to conducting a comparative analysis of known variants of the fictitious domain method for nonlinear Navier-Stokes equations of a viscous incompressible fluid. Along with the classical variants of the fictitious domain method based on extensions by lower and higher coefficients, a family of fictitious domain methods is developed using the general approach, HUM (Hilbert Uniqueness Method) developed by J.-L. Lions. In the first group of methods of fictitious domains, problems arise for numerical solution of equations with strongly varying coefficients and with a poorly conditioned matrix. When using the second group of methods, problems arise with automating the computational process in an area with a complex boundary. This project will build new iterative methods for solving equations with highly variable coefficients. The problems of reducing the problem of the method of fictitious domains to extreme ones and applying the method of conjugate equations are considered. The country's well-known geological exploration company «ECOSERVICE-C» LLP acts as a co-financing organization.The specialists involved in the project have a sufficient number of scientific results to achieve the project goal. They have published works in fundamental and applied areas. The research group will include only those scientists and specialists who are directly involved in solving the stated tasks.
Purpose	Constructing the numerical method for Navier-Stokes equations in complex geometrical domains. Development of numerical method for solving elliptic equation with highly variable coefficients that occurs when using FDM for Navier-Stokes equations. Development of FDM for Navier-Stokes in variational formulation with Lagrange multiplier defined at actual-boundary using the conjugate equations theory.
Objectives	The main task of the project is to solve the difficulties encountered in the numerical solution of the Navier-Stokes equations for a viscous incompressible liquid. The first difficulty is related to the setting of the boundary condition for pressure due to the lack of a physical formulation of the problem. The second difficulty is the complex curved boundaries of the integration domain. To overcome these difficulties, this project will develop effective methods of fictitious domains for numerical solution of the Navier-Stokes equation, the main idea of which is to move from solving problems in areas with a

	curved boundary to solving problems or a sequence of problems in areas whose boundaries would be simpler.
Expected and achieved results	A comparative analysis of two FDM families for the Navier-Stokes equations will be performed. Strictly mathematical problems of approximation, stability and convergence of the FDM auxiliary problem will be studied. Numerical calculations will be performed in a wide range of coefficients of the equation and a small parameter of the FDM on the example of the model problem. A software will be created for numerical solution of the Navier-Stokes system of equations using the fictitious domain method. The software will be created using object oriented programming and a new generation user interface. Thus, as a result of the project, the theoretical research of FDMs for Navier-Stokes equations at the differential and difference levels will be considered, effective algorithms for numerical implementation will be built, software will be developed using modern programming language, the numerical solutions of the problem will be found and presented in graphical form. The results obtained in the project are important, since the Navier-Stokes equations describe the physics of many phenomena and are used to solve scientific questions and technological problems of industrial development on a national and international scale. These equations are used to model weather forecasts, air pollution, complex ocean currents, fluid flows in a pipe, and wing profile flow problems. Navier-Stokes equations are used in designing the streamlined shape of aircraft and automobiles, analyzing and studying blood vessels, and many other processes. In combination with Maxwell's equations, they can be used for modeling and study in magnetic hydrodynamics. The Navier-Stokes equations are also of great scientific interest in a purely mathematical sense, i.e. the problem of existence and smoothness of the solution in the three-dimensional case remains open.
Research team members with their identifiers (Scopus Author ID, Researcher ID, ORCID, if available) and links to relevant profiles	1. Temirbekov Almas Nurlanovich, PhD, Associate Professor, Scopus h-index:5, Web of Science h-index:2, Web of Science ResearcherID: ECD-5970-2022, Scopus Author ID: 56436563100, ORCID: 0000-0002-4157-2799 2. Kasenov Syrym Erkinovich, PhD Associate Professor, Scopus h-index: 5, Web of Science h-index: 2, Web of Science ResearcherID: S-2074-2019, https://orcid.org/0000-0002-0097-1873, Scopus Author ID: 55964589700 3. Temirbekova Laura Nurlanovna, PhD, Scopus h-index: 4, Web of Science h-index: 1, Web of Science ResearcherID: P-7049-2017, https://orcid.org/0000-0003-2456-9974, Scopus Author ID: 55508043100 4. Tamabay Dinara Orazbekkyzy, Master of Sciences, Scopus h-index: 1, Web of Science h-index: 1, https://orcid.org/0000-0001-8315-5849, Web of Science

	ResearcherID: IRU-3078-2023, Scopus Author ID: 58192775000
List of publications with links to them	1 Temirbekov A., Malgazhdarov Y., Tleulessova A., Temirbekova L. Fictitious domain method for the Navier-Stokes equations// Известия НАН РК, серия «физико-математическая». – 2021.-№3(337). – С.128-137. https://doi.org/10.32014/2020.2518-1726.55 (КОКСОН МНВО РК). 2 Kasenov S.E., Temirbekov A.N., Satybaev A. ZH., Temirbekova L.N. Application of the fictitious domain method for ordinary differential equations // Вестник КазНПУ им. Абая, серия «физико-математические науки». – 2021. №2(74). – С.5-12. https://doi.org/10.51889/2021-2.1728-7901.01 (КОКСОН МНВО РК). 3 Temirbekov L.N., Malgazhdarov E.A. Creation and evaluation of the structures grid in curvilinear areas// Вестник КазНУ им. аль-Фараби, серия «математика, механика, информатика». – 2021.-№3(111).–С.122-131. https://doi.org/10.26577/JMMCS.2021.v111.i3.10 (КОКСОН МНВО РК). 4 Temirbekov A., Kasenov S., Temirbekova L. Fictitious domain method for atmosphere boundary layer model. 5th International Conference of Mathematical Sciences (ICMS 2021) 23-27 June 2021, Maltepe University, Istanbul, Turkey. – 2021. – P.97. 5 Темирбеков А. Н., Касенов С. Е. Численная реализация метода фиктивных областей для уравнения эллиптического типа// Вестник НИА РК. –2022.-№3(85). – С.168-181. https://doi.org/10.47533/2020.1606-146X.188 (КОКСОН МНВО). 6 Temirbekov A., Zhaksylykova Z., Malgazhdarov Y., Kasenov S. Application of the fictitious domain method for Navier-Stokes equations. Computers, Materials and Continua. –2022. -Vol.73, N.1.- P.2035–2055. https://doi.org/10.32604/cmc.2022.027830 (Scopus procentile – 78, SJR=0.525, Scopus quartile – Q1, Web of Science quartile - Q2, IF=3.1). 7 Temirbekov A., Altybay A., Temirbekova L., Kasenov S. Development of parallel implementation for the Navier-Stokes equation in doubly connected areas using the fictitious domain method. Eastern-European Journal of Enterprise Technologies. –2022. -Vol. 2, Issue4(116).- P.38–46. https://doi.org/10.15587/1729-4061.2022.254261 (Scopus procentile – 47, SJR= 0.283, Scopus quartile - Q3). 8 Temirbekov A., Kasenov S., Temirbekova L. Development of a computational algorithm for the numerical solution of the Navier-Stokes equations by the fictitious domain method. 6th International Conference of

	Mathematical Sciences (ICMS 2022) 20-24 July 2022, Maltepe University, Istanbul, Turkey. – 2022. – P.60. 9 Temirbekov A., Kasenov S., Temirbekova L. Fictitious domain method for atmosphere boundary layer model. AIP Conference Proceedings Volume 2483, Article number 060009. –2022. https://doi.org/10.1063/5.0115504 (Scopus procentile – 15, SJR= 0.164, Scopus quartile – Q4). 10 Temirbekov A., Malgazhdarov Y., Kasenov S., Temirbekova L. Application of the fictitious domain method for Navier-Stokes equations in natural variables. Proceedings of the 8th international conference on control and optimization with industrial applications (COIA 2022) 24-26 August 2022, Baku, Azerbaijan . – 2022.-Vol. 2.-P.459-461. 11 Temirbekov A.N., Temirbekova L.N., Zhumagulov B.T. Fictitious domain method with the idea of conjugate optimization for non-linear Navier-Stokes equations. Applied and Computational Mathematics. –2023.-Vol. 22, Issue 2.-P.172–188. https://doi.org/10.30546/1683-6154.22.2.2023.172 (Scopus procentile – 98, SJR=1.191, Scopus quartile - Q1, Web of Science quartile – Q1, IF=10). 12 Temirbekov A., Zhumagulov, B. Variational methods for constructing iterative algorithms. AIP Conference Proceedings Volume 2781, Article number 020060. - 2023. https://doi.org/10.1063/5.0144819 (Scopus procentile – 15, SJR= 0.164, Scopus quartile – Q4).
Patents	-